Project Title:

Use a conversion graph to find an estimate for the VAT inclusive price of various items at a mini mart.

Introduction:

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The purpose of this project is to estimate the VAT inclusive price of various items sold at a mini mart without having to resort to actual calculations.
A conversion graph is constructed and placed on an inner wall of the mini mart which is easily accessible to customers.
A customer who does not have a calculator can easily get an estimate of the VAT inclusive price of an item whose cost price is unknown.
The conversion graph could be the size of the large whiteboard, or larger, that is used in schools so that it is easily readable.

The conversion graph will be tested by finding the estimated VAT inclusive price of an item costing

(a) $45
(b) $68
(c) $82

The values will be calculated using a calculator and compared pairwise.

Method of Data Collection:

The following data was collected from a reputable business owner via an interviewer.
The business owner was asked to complete the table below.

Cost Price ($)	VAT Inclusive Price ($)
10.00	
20.00	
30.00	
40.00	
50.00	
60.00	
70.00	
80.00	
90.00	
100.00	

The data was verified by checking with the District Revenue Office in Couva.
The data collected is discrete and ungrouped.

Presentation of Data:

The following data was collected from the reputable business owner and verified by paying a visit to the District Office in Couva.

Cost Price ($)	VAT Inclusive Price ($)
10.00	11.25
20.00	22.50
30.00	33.75
40.00	45.00
50.00	56.25
60.00	67.50
70.00	78.75
80.00	90.00
90.00	101.25
100.00	112.50

The two variables are 'Cost Price' and 'VAT inclusive price' which are both discrete.

A graph of VAT inclusive price versus cost price was drawn on the Cartesian plane, which for now is a sheet of graph paper.

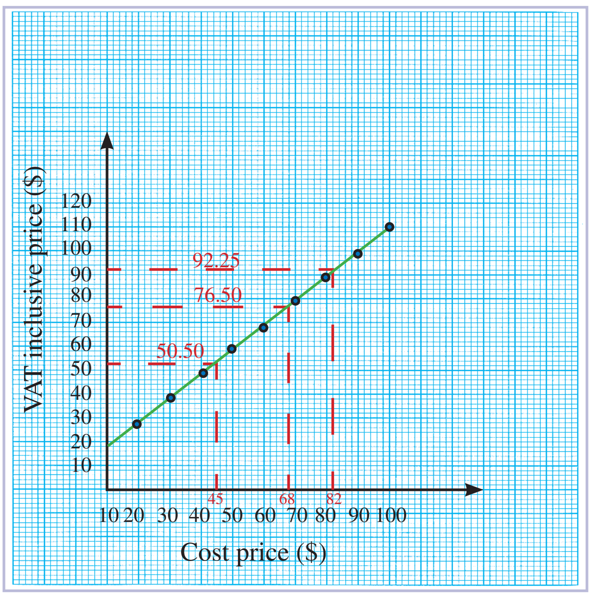
After plotting the ten points a line of best-fit was drawn through the points.
The VAT rate is $12\frac{1}{2}$%.

Later on a grid was drawn on the Cartesian plane which is the wall. The graph was then drawn on the grid for use by customers.

A customer who knows the cost price of an item can then use the graph to interpolate or extrapolate and find the VAT inclusive price of the item.

Analysis of Data:

Below is the conversion graph that was drawn on graph paper. The scales do not allow for extrapolation.



On the wall the axes can be lengthen to cater for more values so that extrapolation will be possible.

- What is the VAT inclusive price of a toy costing $45?

First draw a vertical line from $45 on the horizontal axis which shows the cost price to intersect the conversion graph which is a line. Next draw a horizontal line from the point of intersection to meet the vertical axis which shows the VAT inclusive price. The value of the VAT inclusive price is then read off.

The process of using the plotted point on a graph as described above is known as interpolation.
If the line is produced as expected and used to find the estimated VAT inclusive price of an item, then that process is called extrapolation.

From calculation using a calculator:

The VAT inclusive price of a toy

$= \$45 \times 112\frac{1}{2}\,\%$
$= \$45 \times 112.5/100$
$= \$45 \times 1.125$
$= \$50.63$ (correct to the nearest cent)

Note that: $50.63 - $50.50 = $0.13

The estimated VAT inclusive price found using the conversion graph is $0.13 (or 13 cents) less than the calculated price.
The estimated price is reasonable.

What is the VAT inclusive price of a ball costing $68?

A similar construction is done on the graph as explained previously.

From the construction:

The VAT inclusive price of a ball costing $68 = $76.50

From calculation:

The VAT inclusive price of a ball costing

$68 = $68 × 1.125
= $76.50

Note that: $76.50 - $76.50 = $0

The estimated VAT inclusive price using the conversion graph is exactly the same as the calculated price.

What is the VAT inclusive price of a CD costing $82?

From the construction:
The VAT inclusive price of a CD costing $82 = $92.25

From calculation:

The VAT inclusive price of a CD costing
$82 = $82 × 1.125
= $92.25
Both prices are exactly the same value.

Discussion and Findings:

The VAT inclusive price found using the conversion graph is not always equal to the price found using a calculator, which is always accurate, at least to the nearest cent.

Using the conversion graph to find the VAT inclusive price of an item whose cost price is known is not 100% accurate always, but it is good enough as an estimate correct to the nearest dollar, at least, since sometimes it is out by a few cents.

Conclusion:

The conversion graph method of finding the VAT inclusive price of an item is good enough for a mini mart where a customer is more interested in finding an estimate of the VAT inclusive price for an item.

Even if a calculator is used to calculate the VAT inclusive price of an item sometimes the value has to be rounded correct to the nearest cent if it is not an exact value as in the case of the toy costing $45.

The method of using a conversion graph to find the VAT inclusive price is reasonable enough for a mini mart customer to use on a daily basis to find a quick estimate of the value.

The accuracy of the VAT inclusive price found from a conversion graph can be investigated by changing the scale on the vertical VAT inclusive axis when accuracy to the nearest cent may be required. This fact can be realized by using a large Cartesian plane like a whiteboard instead of a small sheet of graph paper. The grid drawn on a whiteboard or on the wall in the mini mart can accommodate the scale required for values to the nearest cent.

References:
Toolsie, Raymond *(1996), Mathematics: A Complete Course with CXC Questions, Caribbean Educational Publishers.*

Project Title:

To calculate the point (x, y) where two running boys reach a bus.

Introduction:

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The purpose of this project is to calculate the point (x, y), using a matrix method, where two running boys meet a bus.

A pair of linear equations will be written as a matrix equation and then solved with the use of an inverse matrix and multiplication of $(2\times2) \times (2\times1)$ matrices.

Method of Data Collection:

The Records from a department states that Michael is running along a path defined by the linear equation

$$5x + 2y = 34$$

It is also given that Nigel is running along a path given by the linear equation

$$2x - 3y = -13$$

$x \in R$ and $y \in R$ so both x and y are real numbers.

The data represented by the linear equations is continuous but the point (x, y) is discrete.

The x axis and the y axis are two axes at right-angles to each other.

If two straight lines are drawn on graph paper to represent the two linear equations, then the point (x, y) is the point of intersection of the two lines.

Presentation of Data:

The two linear equations involved are

$$5x+2y=34$$
$$2x-3y=-13$$

The two equations are to be solved simultaneously, using a matrix method which uses an inverse matrix.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a matrix,

then $A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} a & -b \\ -c & d \end{pmatrix}$ is the inverse matrix of A.

If the matrix equation is in the form

$AX = B$ then
$X = A^{-1} B$

And the matrix

$X = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow$ = the point (x, y)

The two variables 'x' and 'y' are both continuous.

Analysis of Data:

The pair of linear equation is

$$5x+2y=34$$
$$2x-3y = -13$$

The linear equation can be written as the matrix equation

$$\begin{pmatrix}5 & 2\\2 & -3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}34\\-13\end{pmatrix}$$

It is in the form

$$AX = B$$

So $X = A^{-1} B$

Thus $A = \begin{pmatrix}5 & 2\\2 & -3\end{pmatrix}$

And $A^{-1} = \frac{1}{5\times[-3]-2\times 2}\begin{pmatrix}-3 & -2\\-2 & 5\end{pmatrix}$

$$= \frac{1}{-15-4}\begin{pmatrix}-3 & -2\\-2 & 5\end{pmatrix}$$

$$= \frac{1}{-19}\begin{pmatrix}-3 & -2\\-2 & 5\end{pmatrix}$$

$$= \frac{1}{-19}\begin{pmatrix}-3 & -2\\-2 & 5\end{pmatrix}$$

So X $= \frac{1}{19}\begin{pmatrix}3 & 2\\2 & -5\end{pmatrix}\begin{pmatrix}34\\-13\end{pmatrix}$

$$= \frac{1}{19}\begin{pmatrix}3\times 34+2\times[-13]\\2\times 34-5\times[-13]\end{pmatrix}$$

$$= \frac{1}{19}\begin{pmatrix}102-26\\68+65\end{pmatrix}$$

$$= \frac{1}{19}\begin{pmatrix}76\\133\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{9}\times 76\\\frac{1}{9}\times 133\end{pmatrix}$$

i.e. $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\7\end{pmatrix}$

Hence $x = 4$ when $y = 7$

Hence, the point(x,y) where the two running boys reach the bus is (4,7).

The solution can be checked by substituting the value for x and y into the left hand expression of each of the linear equations.

When $x = 4, y=7$: $5x+2y = 5\times4+2\times7$
$= 20+14$
$5x+2y = 34$ which is correct
$2x-3y = 2(4) - 3(7)$
$= 8-21$
$2x-3y = -13$ which is correct Hence, the solution is correct.

Hence, the solution is correct.

Discussion and Findings:

It is clear that the two running boys would reach the bus at the point(4,7). It is an accurate solution verified by substituting the value for x and the value for y into expression in the left hand side of each equation and obtaining the constant terms on the right hand side.

The method of solving the pair of linear equations simultaneously, using the matrix method, which is based on finding an inverse of a matrix and the multiplication of matrices is very accurate.

Conclusion:

The method of using matrices to find the point where the two running boys meet is very accurate and relatively easy to perform.

References:

Toolsie, Raymond *(1996), Mathematics: A Complete Course with CXC Questions, Caribbean Educational Publishers.*

Project Title:

Doctor to prescribe two capsules so that the patient receive a daily minimum number of units of three medications at a minimum cost to the patient.

Introduction:

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The purpose of this project is to determine how many of two different capsules containing three medications each a doctor must prescribe, so that the patient receives a minimum number of units of each medication, at a minimum cost to the patient.

Three linear inequalities will be used to represent the minimum daily number of units of the three medication to the taken by the patient.

The regions of the linear inequalities will then be represented on a graph and the common region will then be recognized as the overlapping of the three shaded region.

A vertex or vertices in the common region will offer a solution to the problem.

An equation will be formed to represent the daily cost of the capsules to the patient.

The solution vertex is used with the equation to give the minimum daily cost of the capsules used by the patient.

Method of Data Collection:

The problem was outlined by the doctor as follows:
The table shows the medication on each capsule, in number of units per capsule.

Capsule	Medication A	Medication B	Medication C
X Y	7 14	15 10	3 15

The patient must receive daily, at least 56 units of Medication A, 60 units of Medication B and 30 units of Medication C.

Capsule X costs \$3.70 per capsule and
Capsule Y costs \$5.90 per capsule

Let the number of Capsule X used $= x$
and the number of Capsule Y used $= y$

The problem can be represented by three inequalities and are equation as follows:

$7x+14y \geq 56$ 1.
$15x+10y \geq 60$ 2.
$3x+15y \geq 30$ 3.

The cost of x Capsule X
and y Capsule Y, $C = \$ (3.70x + 5.90y)$

The data is discrete data.
$x \in N$ and $y \in N$

Presentation of Data:

		$7x+14y$	≥ 56	(1)
1.	$\div 7$:	$x+2y$	≥ 8	(4)
		$15x+10y$	≥ 60	(2)
2.	$\div 5$:	$3x+2y$	≥ 12	(5)
		$3x+15y$	≥ 30	(3)
3.	$\div 3$:	$x+5y$	≥ 10	(6)

The reduced inequalities are:

$x+2y$	≥ 8	(4)
$3x+2y$	≥ 12	(5)
$x+5y$	≥ 10	(6)

From (4):	$x+2y$	≥ 8	
so	$2y$	$\geq -x+8$	
	y	$\geq \frac{-x+8}{2}$	(7)
From (5):	$3x+2y$	≥ 12	
so	$2y$	$\geq -3x+12$	
	y	$\geq \frac{-3x+12}{2}$	(8)
From (6):	$x+5y$	≥ 10	
so	$5y$	$\geq -x+12$	
	y	$\geq \frac{-x+10}{5}$	(9)

The two variables are 'number of Capsule X' and the 'number of Capsule Y' which are both discrete.

From the last three representations of the inequalities we get the border equations:

$y = \frac{-x+8}{2}$ (10)

$y = \frac{-3x+12}{2}$ (11)

$y = \frac{-x+10}{5}$ (12)

The following table of values were then constructed for the three equations:

x	0	2	4
$-x+8$	8	6	4
y	4	3	2

$y = \frac{-x+8}{2}$ for the domain $0 \leq x \leq 4$

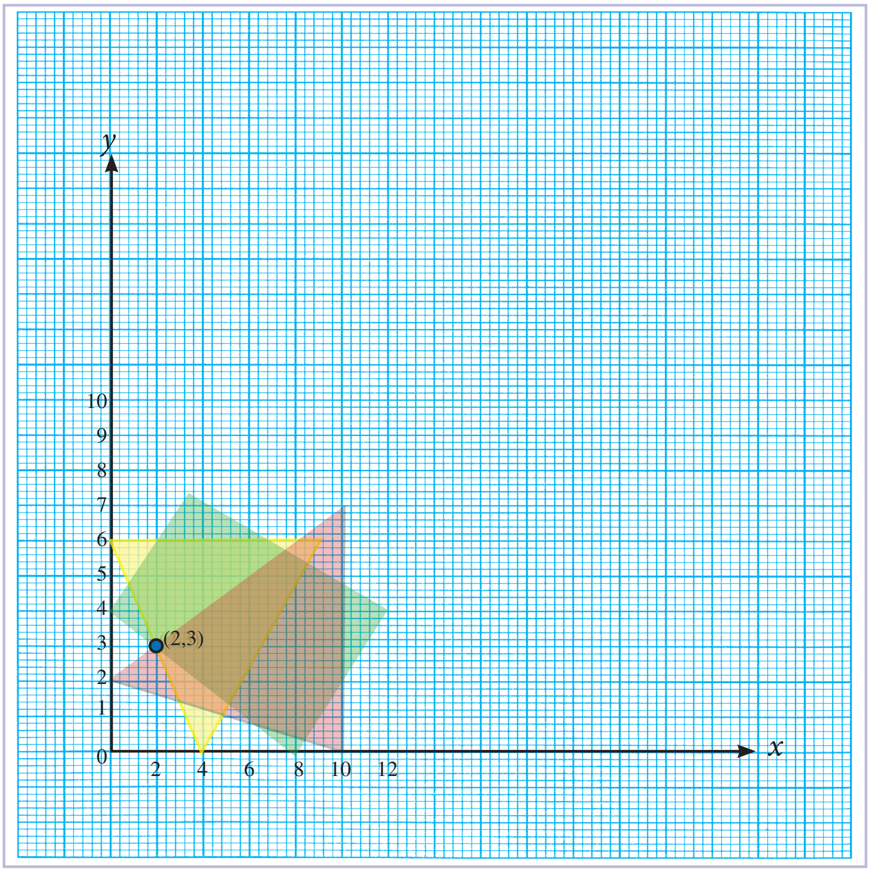
x	0	2	4
$-3x + 12$	12	6	0
y	6	3	0

$y = \frac{-3x+12}{2}$ for the domain $0 \leq x \leq 4$

x	0	5	10
$-x + 10$	10	5	0
y	2	1	0

$y = \frac{-x+10}{5}$ for the domain $0 \ \leq x10$

Using the three tables of values three lines that represent the three linear equations were drawn on graph paper with the same scales and axes as shown.



The region represented by each of the three inequalities were then shaded as shown on the graph.
The only viable vertex in the common region that is, where the shading over laps is(2,3).
The vertex (2,3) means that x = 2 when y = 3.

Analysis of Data:

The only common vertex in the common region is(▪(2,3)).
This implies that the conditions represented by the three inequalities is satisfied by the vertex(2,3).

Hence, the doctor must recommend 2 Capsule X and 3 Capsule Y for the patient to take daily.
The cost of 2 Capsule X and 3 Capsule Y, C min $= \$ (3.70 \times 5.90 \times 3)$
$= \$ (7.40 + 17.70)$
$= \$ 25.10$

Hence, the minimum daily cost to the patient is \$ 25.10.

Discussion of Findings:

When x=2 and y = 3:

$x+2y = 2 + 2 \times 3 = 2 + 6 = 8$

so $x+2y \geq 8$

$3x+2y = 3 \times 2 + 2 \times 3 = 6 + 6 = 12$

so $3x+2y \geq 12$

$x+5y = 2 + 5 \times 3 = 2 + 15 = 17$

so $x+5y \geq 10$

Clearly, all three inequalities are satisfied by the vertex(2,3).

Conclusion:

If the doctor prescribes 2 Capsule X and 3 Capsule Y daily, then he would have satisfied all the conditions specified by the three inequalities, which means that the patient will receive the minimum number of units of each of the three medications.

The cost of \$25.10 daily seems to be a reasonable cost for the patient to bear.

References:
Toolsie, Raymond *(1996), Mathematics: A Complete Course with CXC Questions, Caribbean Educational Publishers.*